Seating Round a Table

Mathematics | JEE Main & Advanced

Don't Memorize. Visualize.

> n people can sit round a table in $(n-1)!$ ways, not $n!$. Moving everyone one seat round gives the same seating, because everyone keeps the same neighbours on the same sides.

FACTORIAL

$n! = n \times (n-1) \times \cdots \times 1$: the number of ways to put n people in a row.

$4! = 24$, $3! = 6$.

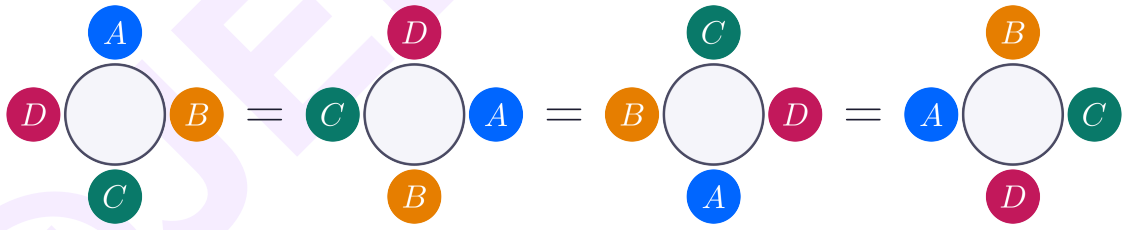
CIRCULAR ARRANGEMENT

An order round a circle, where only the positions of the people relative to each other matter, not which chair.

No first seat, no last seat.

Why we divide by n

In a row, 4 people sit in $4! = 24$ ways. Round a table, move everyone one seat clockwise: the picture changes, but A still has B on one side and D on the other, and so on for everyone. It is the same seating. Doing it four times brings everyone back:



So each round-table seating appears 4 times among the 24 row orders:

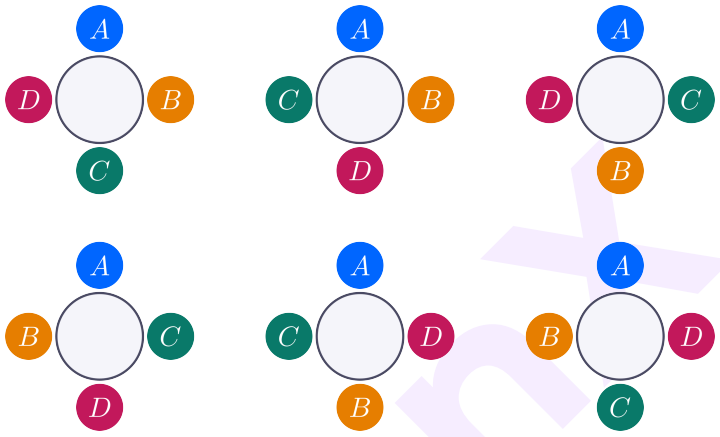
$$\text{seatings} = \frac{4!}{4} = 3! = 6, \qquad \text{in general } \frac{n!}{n} = (n-1)!.$$

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Or fix one person

A second way to see it: since turning changes nothing, let A take any seat and count from there. The other three fill the three seats left, in order clockwise from A: $3! = 6$ ways. Here are all six, A at the top.

Each is different from the others: in each, someone has a different neighbour, or a neighbour on the other side.



CAREFUL

Clockwise and anticlockwise orders are **different** seatings: in ABCD (clockwise) B sits on one side of A, in ADCB on the other. But for a necklace or a garland, which can be turned over, the mirror image is the same object, so there are only $\frac{(n-1)!}{2}$ ways (for $n \geq 3$).

When the chairs are numbered

The $(n-1)!$ rule assumes only who sits next to whom matters. If the chairs are numbered (or a seat is marked, like the head of the table), moving everyone one seat round *does* change the seating: each person is now in a different chair. Then all $n!$ orders are different. Read the question for this.

Solved example

In how many ways can 6 people sit round a table? In how many of them do two friends, P and Q, sit together?

All seatings. $(6-1)! = 5! = 120$.

P and Q together. Tie them into one block: 5 units round the table, $(5-1)! = 24$ ways, and inside the block P, Q can swap: $24 \times 2 = 48$.

120; 48

Key formulas

n people in a row: $n!$	n people round a table: $(n-1)!$
n beads on a necklace: $\frac{(n-1)!}{2}$	two together (round a table): $(n-2)! \times 2$

Practice set

Easy to hard. The answers start on the next page.

Q1. **Quiz from the Short.** In how many ways can 5 people sit round a table? (A) 120 (B) 60 (C) 24.

Q2. How many different necklaces can be made from 7 different beads?

Q3. 5 people sit round a table. In how many seatings are A and B together?

Q4. In how many of those seatings are A and B *not* together?

Q5. 4 men and 4 women sit round a table, men and women alternating. How many seatings?

Q6. 8 people sit round a table. In how many seatings are two particular people not next to each other?

Watch the animated lesson on YouTube

Seating Round a Table (Short)

Scan the code to open the JEEVisionX channel on YouTube. This lesson is in the JEE Concepts Visualization Shorts playlist.

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Answers

Q1

(C) 24

$(5-1)! = 4! = 24$. Fix one person's seat; the other 4 fill the rest in $4!$ ways. $5! = 120$ counts each seating 5 times, once for each turn of the table.

Q2

360

$\frac{(7-1)!}{2} = \frac{720}{2}$: a necklace can be turned over.

Q3

12

A and B as one block: $(4-1)! = 6$, times 2 for their order.

Q4

12

All seatings minus together: $24 - 12$.

Q5

144

Seat the men: $(4-1)! = 6$. The 4 gaps between them take the women in $4! = 24$ ways: 6×24.

Q6

3600

All: $7! = 5040$. Together: $6! \times 2 = 1440$. Not together: $5040 - 1440$.

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