Half-Life

Physics (JEE Advanced) and Chemistry (JEE Main & Advanced)

Don't Memorize. Visualize.

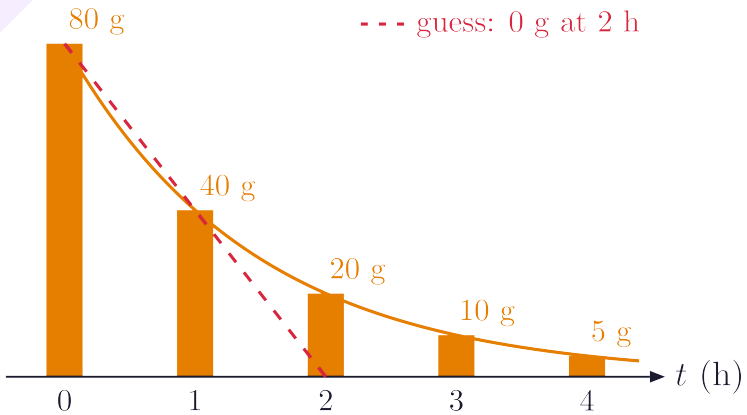
> Each half-life halves what is *left*, not what you started with. After two half-lives a quarter remains, after three an eighth: $N = \dfrac{N_0}{2^n}$ after n half-lives.

HALF-LIFE $T_{1/2}$
The time in which half of the nuclei present decay. It is the same whatever amount you start with.
For a sample, a reaction, a drug in the blood.

DECAY CONSTANT λ
The chance per unit time that one nucleus decays. Larger λ, faster decay.
$T_{1/2} = \dfrac{\ln 2}{\lambda} \approx \dfrac{0.693}{\lambda}$.

Halving what is left

Take 80 g of a radioactive substance with a half-life of 1 hour. In the first hour half of it decays: 40 g left. In the next hour half of *what is left* decays, that is 20 g, not another 40 g: 20 g left, not zero. Then 10 g, 5 g, and so on. The amount that decays in each hour gets smaller, because there is less left to decay.
The red line is the common guess: 40 g in each hour, so nothing left at 2 hours. It agrees with the truth at 1 hour and fails after that. Decay is random for each nucleus, but 80 g holds more than 10^{23} nuclei, so the halving holds very closely.



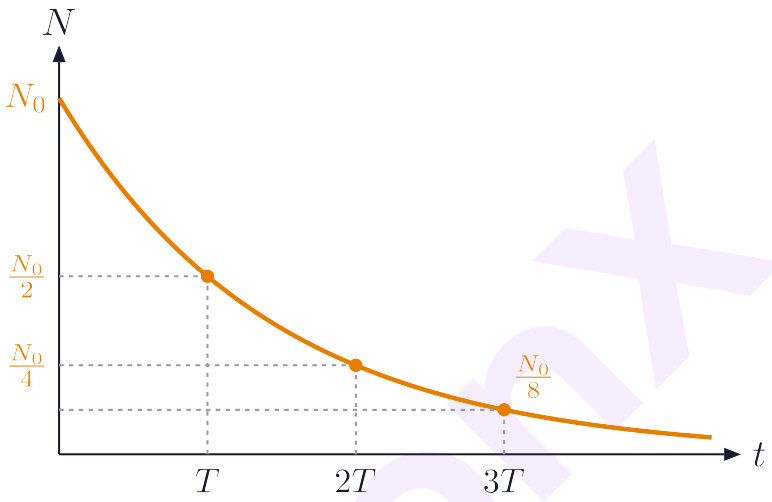
The law

$$N = N_0\, e^{-\lambda t} = N_0 \left(\frac{1}{2}\right)^n, \qquad n = \frac{t}{T_{1/2}} \text{ (the number of half-lives).}$$

The decay curve

Plot N against t. Every step of one half-life cuts the height in half, so the curve falls fast at first and then more and more slowly. The law never gives zero: each half-life leaves half of what was there.

The same law runs a **first-order reaction** in chemistry: the concentration halves every $t_{1/2} = \frac{0.693}{k}$, whatever it started at.



CAREFUL

"Half decays in 1 hour, so all of it is gone in 2 hours" treats the second half as decaying at the same rate as the first. But in the second hour only half of what is *left* decays: $\frac{N_0}{2} \to \frac{N_0}{4}$. After 2 hours a quarter is still there.

Solved example

A sample of 80 g has a half-life of 5 days. How much is left after 15 days?

Count half-lives. $n = \frac{15}{5} = 3$.

Halve three times. $80 \to 40 \to 20 \to 10$, that is $80 \times \left(\frac{1}{2}\right)^3$.

10 g

Key formulas

$$N = N_0 \left(\tfrac{1}{2}\right)^n = \frac{N_0}{2^n} \qquad n = \frac{t}{T_{1/2}}$$

$$N = N_0\, e^{-\lambda t} \qquad T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

$$\text{fraction decayed} = 1 - \frac{1}{2^n} \qquad \text{first-order reaction: } t_{1/2} = \frac{0.693}{k}$$

Practice set

Easy to hard. The answers start on the next page.

Q1. **Quiz from the Short.** A sample has a half-life of 2 days. After 6 days, the fraction left is: (A) $\frac{1}{6}$ (B) $\frac{1}{8}$ (C) 0.

Q2. A sample has a half-life of 2 hours. How long until only $\frac{1}{8}$ of it is left?

Q3. What fraction of a sample has decayed after 3 half-lives?

Q4. A first-order reaction has $k = 0.0693\ \text{min}^{-1}$. Find its half-life.

Q5. What fraction of a sample is left after 2.5 half-lives?

Q6. Samples A ($T_{1/2} = 1$ h) and B ($T_{1/2} = 2$ h) start with the same number of nuclei. Find $\frac{N_A}{N_B}$ after 4 hours.

Watch the animated lesson on YouTube

Half-Life (Short)

Scan the code to open the JEEVisionX channel on YouTube. This lesson is in the JEE Concepts Visualization Shorts playlist.

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Answers

Q1

(B) $\frac{1}{8}$

6 days are 3 half-lives: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. Each half-life halves what is left, so after 6 days $\frac{1}{8}$ is still there, not 0.

Q2

6 hours

$\frac{1}{8} = \left(\frac{1}{2}\right)^3$: three half-lives of 2 hours.

Q3

$\frac{7}{8}$

Left: $\frac{1}{8}$. Decayed: $1 - \frac{1}{8}$.

Q4

10 min

$t_{1/2} = \frac{0.693}{0.0693}$.

Q5

about 0.177

$\left(\frac{1}{2}\right)^{2.5} = \frac{1}{4\sqrt{2}}$. A half-life count need not be a whole number.

Q6

$\frac{1}{4}$

A: 4 half-lives, $\frac{1}{16}$. B: 2 half-lives, $\frac{1}{4}$. Ratio $\frac{1}{16} \div \frac{1}{4}$.

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