Range of a Projectile

Physics | JEE Main & Advanced

Don't Memorize. Visualize.

> Thrown at the same speed from level ground (no air resistance), the angles θ and $90^\circ - \theta$ land at the same distance. The range depends on $\sin 2\theta$, and $\sin 2\theta = \sin(180^\circ - 2\theta)$.

RANGE

The horizontal distance a projectile covers before it comes back to the height it was thrown from.

Written R.

COMPLEMENTARY ANGLES

Two angles that add up to 90°, like 30° and 60°, or 15° and 75°.

θ and $90^\circ - \theta$.

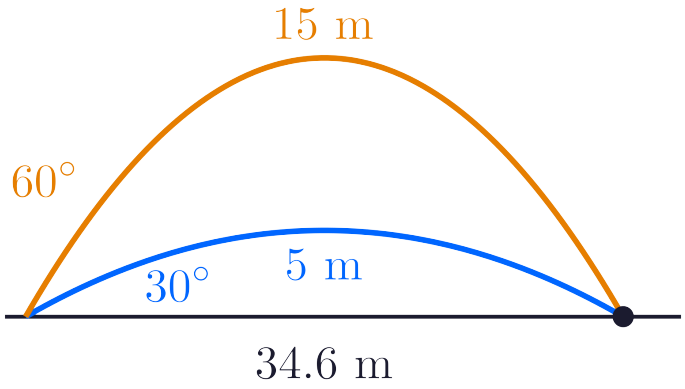
Where the range formula comes from

Launch at speed u, angle θ. Sideways the speed $u\cos\theta$ never changes; upwards it starts at $u\sin\theta$ and gravity slows it.

Time, height, range

$$T = \frac{2u\sin\theta}{g}, \qquad H = \frac{u^2\sin^2\theta}{2g},$$

$$R = u\cos\theta \times T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}.$$



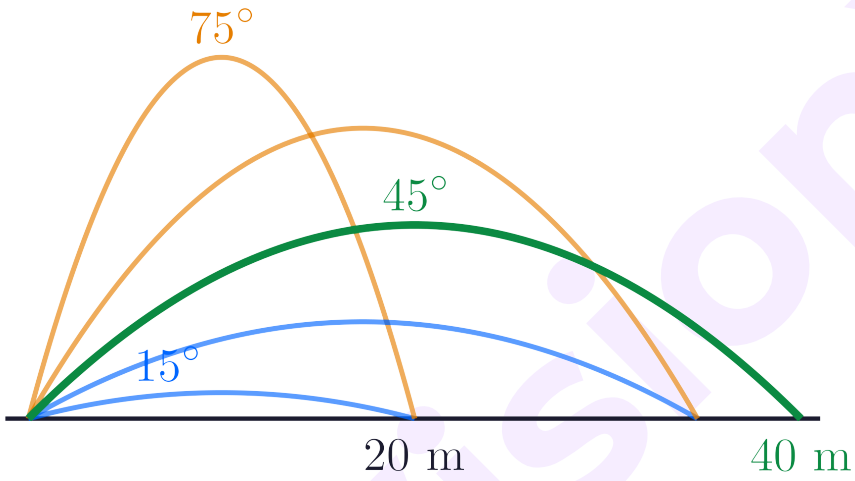
With $u = 20$ m/s and $g = 10\ \mathrm{m\,s^{-2}}$: 30° gives $R = 40\sin 60^\circ = 20\sqrt{3} \approx 34.6$ m, and so does 60°.

Why the pair lands together

For the angle $90° - \theta$,

$$\sin 2(90° - \theta) = \sin(180° - 2\theta) = \sin 2\theta,$$

so R is the same. The largest range is at $\theta = 45°$, where $\sin 2\theta = 1$: $R_{\max} = \frac{u^2}{g} = 40$ m at 20 m/s. Every other range is reached by two angles, one on each side of $45°$: below, $15°$ and $75°$ land together at 20 m.



CAREFUL

Same range does **not** mean the same flight. The higher angle climbs higher and stays up longer: at 20 m/s, $30°$ rises to 5 m in 2 s of flight, $60°$ to 15 m in $2\sqrt{3} \approx 3.46$ s. And the rule needs level ground and no air: thrown from a cliff, or with drag, the two ranges differ.

Solved example

At 20 m/s, a ball thrown at $20°$ lands 25.7 m away. At what other angle does a ball thrown at the same speed land at the same point?

Pair it. The complementary angle: $90° - 20° = 70°$.

Check. $R = 40 \sin 40° \approx 25.7$ m, and $40 \sin 140° = 40 \sin 40°$.

$70°$

Key formulas

$$T = \frac{2u \sin\theta}{g} \qquad H = \frac{u^2 \sin^2\theta}{2g} \qquad R = \frac{u^2 \sin 2\theta}{g}$$

$$R(\theta) = R(90° - \theta) \qquad R_{\max} = \frac{u^2}{g}\ (\theta = 45°) \qquad T_1 T_2 = \frac{2R}{g}\ \text{(a pair)}$$

Practice set

Easy to hard. Take $g = 10\ \mathrm{m\,s^{-2}}$, level ground, no air resistance. The answers start on the next page.

Q1. Quiz from the Short. At the same speed, on level ground with no air, which angle gives the same range as $20°$? (A) $40°$ (B) $80°$ (C) $70°$.

Q2. A ball is thrown at 10 m/s at $30°$. Find its range.

Q3. Find the greatest range for a launch speed of 30 m/s, and the angle that gives it.

Q4. A ball thrown at 20 m/s lands 20 m away. Find the two possible angles.

Q5. Two balls are thrown at the same speed at $30°$ and $60°$. Find the ratio of their greatest heights.

Q6. For a pair of angles that give the same range R, show that the product of the two times of flight is $\frac{2R}{g}$. Check it for 20 m/s at $30°$ and $60°$.

Watch the animated lesson on YouTube

Range of a Projectile (Short)

Scan the code to open the JEEVisionX channel on YouTube. This lesson is in the JEE Concepts Visualization Shorts playlist.

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Answers

Q1

(C) 70°

$R = \dfrac{u^2 \sin 2\theta}{g}$, and $\sin 140^\circ = \sin 40^\circ$, so 70° and 20° give the same range: the two angles add to 90°. Doubling the angle to 40° gives $\sin 80^\circ$, a longer range.

Q2

$5\sqrt{3} \approx 8.66$ **m**

$R = \frac{10^2 \sin 60^\circ}{10} = 10 \times \frac{\sqrt{3}}{2}$.

Q3

90 m, at 45°

$R_{\max} = \frac{u^2}{g} = \frac{900}{10}$.

Q4

15° **or** 75°

$20 = \frac{400 \sin 2\theta}{10}$ gives $\sin 2\theta = \frac{1}{2}$, so $2\theta = 30^\circ$ or 150°.

Q5

$1 : 3$

$H \propto \sin^2\theta$: $\frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \frac{1}{4} \div \frac{3}{4} = \frac{1}{3}$. (5 m and 15 m at 20 m/s.)

Q6

$T_1 T_2 = \frac{2R}{g}$

$T_1 T_2 = \frac{2u \sin\theta}{g} \cdot \frac{2u \cos\theta}{g} = \frac{2}{g} \cdot \frac{u^2 \sin 2\theta}{g}$. Check: $2 \times 2\sqrt{3} = 4\sqrt{3} \approx 6.93$, and $\frac{2 \times 20\sqrt{3}}{10}$ is the same.

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