A Capacitor at Switch-On

Physics | JEE Main & Advanced

Don't Memorize. Visualize.

In a DC circuit, an **uncharged** capacitor acts like a plain wire just after switch-on, and like an open circuit after a long time.

CAPACITOR

Two plates with a gap. Charge q gathers on them, and the voltage across it is $V_C = \frac{q}{C}$.

No charge crosses the gap.

TIME CONSTANT

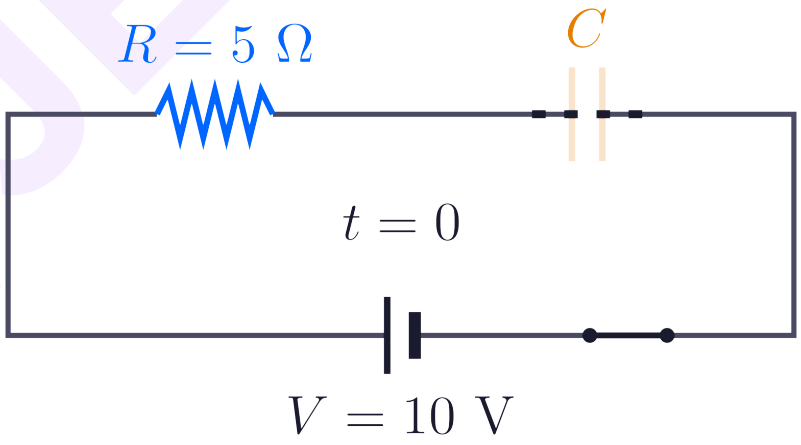
RC, the time scale of charging. After one RC the current has fallen to $e^{-1} \approx 37\%$ of its start.

Units: ohm $\times$ farad $=$ second.

Just after switch-on: like a wire

A capacitor C, a resistor $R = 5\ \Omega$ and a 10 V battery in one loop. The capacitor starts **uncharged**: $q = 0$, so $V_C = \frac{q}{C} = 0$. Just after the switch closes, the capacitor still has no voltage across it, the same as a plain wire (dashed in the figure). All 10 V is across R:

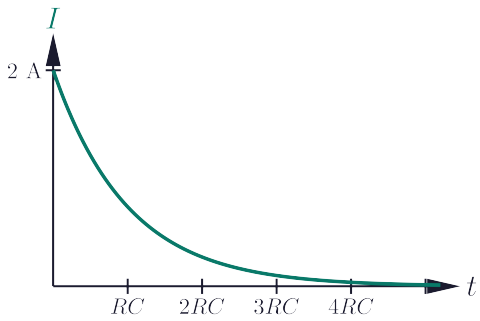
$$I = \frac{V}{R} = \frac{10}{5} = 2\ \text{A}.$$

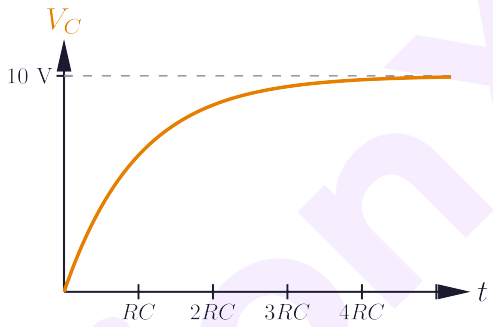


As it charges, the current falls

Charge builds up on the plates, V_C rises, and less voltage is left across R, so the current falls. For an uncharged capacitor charging through R from a battery V:

$$I = \frac{V}{R}\,e^{-t/RC}, \qquad V_C = V\left(1 - e^{-t/RC}\right).$$





t	0	RC	$2RC$	$3RC$
I (A)	2	0.74	0.27	0.10
V_C (V)	0	6.32	8.65	9.50

After a long time: like an open circuit

The current keeps falling towards 0, and V_C rises towards the battery's 10 V. With (almost) no current, the capacitor acts like a **gap in the circuit**: an open circuit. In the DC steady state, no current flows through a capacitor.

CAREFUL

Never exactly zero. At any finite time $e^{-t/RC} > 0$, so the current is never exactly 0 and V_C never exactly 10 V; they *tend* to those values. After $5RC$ the current is under 1% of its start ($e^{-5} \approx 0.0067$), which is why "a long time" means a few RC.
Only an uncharged capacitor acts like a wire at switch-on. One that already holds a voltage keeps it at the first instant: it acts like a battery of that voltage, not a wire.

Solved example

A 12 V battery, a 4 Ω resistor and an uncharged capacitor are joined in one loop with a switch. Find the current and V_C just after the switch closes, and what they tend to after a long time.

Just after: $V_C = 0$, so $I = \dfrac{12}{4} = 3$ A. **Long after:** $I \to 0$ and $V_C \to 12$ V.

3 A and 0 V; then 0 A and 12 V

Key formulas

$$I = \frac{V}{R}\,e^{-t/RC} \qquad V_C = V\left(1 - e^{-t/RC}\right) \qquad \tau = RC$$

$$t = 0\text{: } I = \frac{V}{R},\ V_C = 0 \qquad t \gg RC\text{: } I \to 0,\ V_C \to V$$

Practice set

Easy to hard. Every capacitor starts uncharged, and every battery is ideal. The answers start on the next page.

Q1. Quiz from the Short. In the Short's circuit (10 V battery, $R = 5\ \Omega$, uncharged C), long after switch-on, the voltage across R is about: (A) 0 V (B) 5 V (C) 10 V.

Q2. A 20 V battery, a 10 Ω resistor and a capacitor are joined in one loop. Find the current just after the switch closes.

Q3. In the circuit of Q2, find the current at $t = RC$.

Q4. Find the time constant for $R = 2\ \mathrm{k}\Omega$ and $C = 5\ \mu\mathrm{F}$.

Q5. A capacitor charges through R from a battery V. At what time is $V_C = \dfrac{V}{2}$?

Q6. A 12 V battery is joined in series with $R_1 = 4\ \Omega$ and then a pair in parallel: a resistor $R_2 = 4\ \Omega$ and a capacitor C. Find the battery current just after switch-on and after a long time, and the final V_C.

Watch the animated lesson on YouTube

A Capacitor at Switch-On (Short)

Scan the code to open the JEEVisionX channel on YouTube. This lesson is in the JEE Concepts Visualization Shorts playlist.

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Answers

Q1

(A) about 0 V

Long after switch-on the current tends to 0, so $V_R = IR = 10\,e^{-t/RC}$ V tends to 0. At every moment $V_R + V_C = 10$ V, and now V_C is close to 10 V. 10 V is V_R just after switch-on, the trap; 5 V holds only at $t = RC \ln 2 \approx 0.69\,RC$.

Q2

2 A

$V_C = 0$ at first, so $I = \dfrac{20}{10}$.

Q3

about 0.74 A

$I = 2\,e^{-1} \approx 0.736$ A, about 37% of the start.

Q4

0.01 s (10 ms)

$RC = 2000 \times 5 \times 10^{-6}$ s.

Q5

$t = RC \ln 2 \approx 0.69\,RC$

$V\left(1 - e^{-t/RC}\right) = \dfrac{V}{2}$ gives $e^{-t/RC} = \dfrac{1}{2}$.

Q6

3 A; then 1.5 A; $V_C = 6$ V

Just after, C acts like a wire and shorts R_2: $I = \dfrac{12}{4}$. Long after, no current through C: $I = \dfrac{12}{4+4}$, and V_C equals the voltage across R_2, 1.5×4.

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